

Code No: 181AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, January/February - 2025

MATRICES AND CALCULUS

(Common to EEE, CSE, IT, CSIT, CSE(CS), CSE(DS), CSD)

Time: 3 Hours

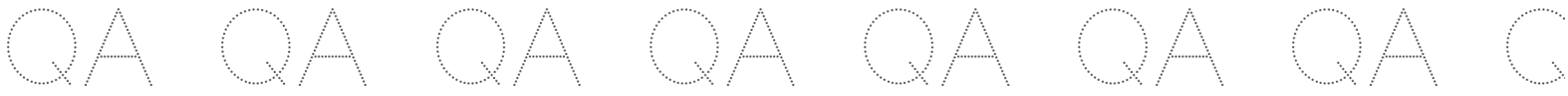
Max. Marks: 60

Note: This question paper contains two parts A and B.i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

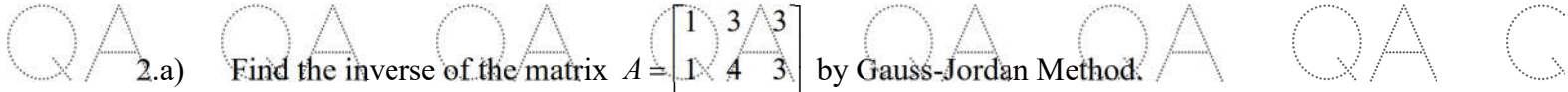
PART- A**(10 Marks)**

- 1.a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$. [1]
- b) Write the condition for the consistency of linear systems $AX = B$, consisting of m linear equations involving n unknowns. [1]
- c) If $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$, then find the Eigen values of A^2 . [1]
- d) State the Cayley-Hamilton Theorem. [1]
- e) Find c of Lagrange's mean value theorem for $f(x) = e^x$ in $[0, 1]$. [1]
- f) Prove that $\int_0^{\infty} e^{-x^2} x^{2n-1} dx = \frac{1}{2} \Gamma(n)$. [1]
- g) Find the first order partial derivatives of $z = x^3 + y^3 - 3axy$. [1]
- h) If $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$. [1]
- i) Evaluate $\int_0^1 \int_0^1 \frac{xdy}{\sqrt{(1-x^2)(1-y^2)}}$. [1]
- j) Evaluate $\int_0^1 \int_1^2 \int_2^3 (xyz) dz dy dx$. [1]



PART - B

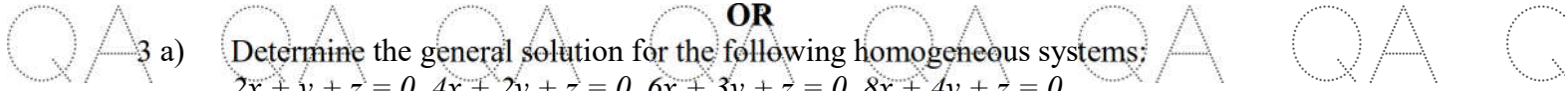
(50 Marks)



2.a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan Method.

b) Discuss the consistency of the following system of equations
 $2x + 3y + 4z = 11$, $x + 5y + 7z = 15$, $3x + 11y + 13z = 25$. If found consistent, solve it.

[5+5]

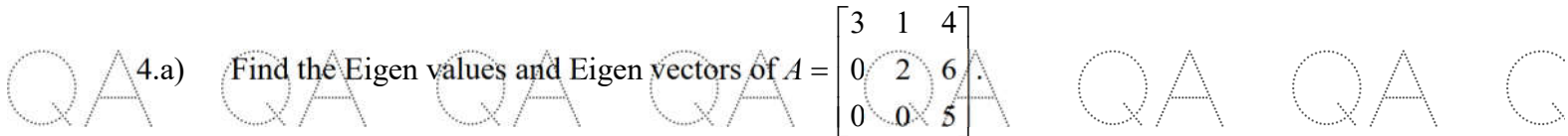


OR

3 a) Determine the general solution for the following homogeneous systems:
 $2x + y + z = 0$, $4x + 2y + z = 0$, $6x + 3y + z = 0$, $8x + 4y + z = 0$.

b) Solve the following system by Gauss-Seidel method:
 $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$, $3x + 8y + 29z = 71$.

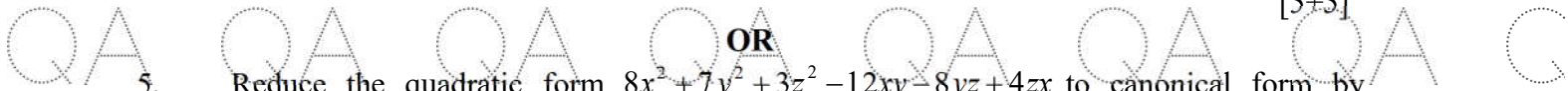
[6+4]



4.a) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

b) Using Cayley-Hamilton theorem, obtain the inverse of the matrix $\begin{bmatrix} -1 & -1 & -2 \\ 8 & -11 & -8 \\ -10 & 11 & 7 \end{bmatrix}$.

[5+5]

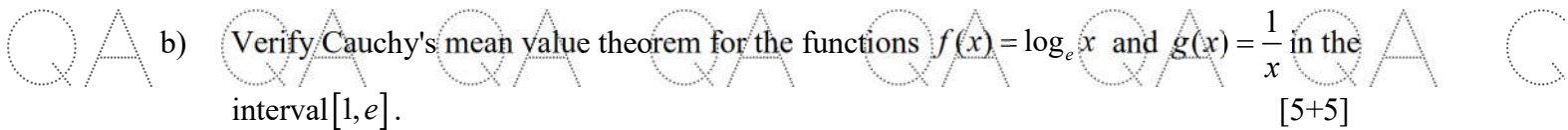


OR

5. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ to canonical form by orthogonal transformation. Determine the index, signature and nature of the quadratic form.

[10]

6 a) Verify Rolle's theorem for the function $f(x) = (x+2)^3(x-3)^4$ in $[-2, 3]$.



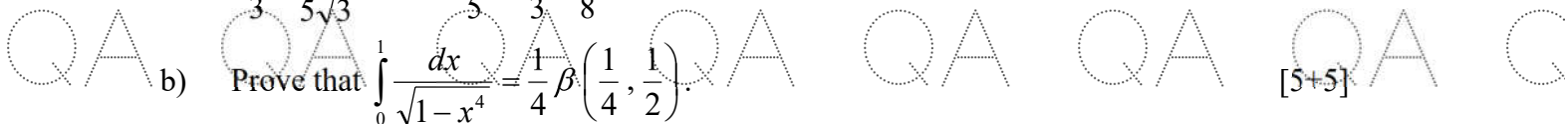
b) Verify Cauchy's mean value theorem for the functions $f(x) = \log_e x$ and $g(x) = \frac{1}{x}$ in the interval $[1, e]$.

[5+5]

OR

7 a) Using the Lagrange mean value theorem, show that

$$\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1} \frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$$



b) Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{1}{4} \beta\left(\frac{1}{4}, \frac{1}{2}\right)$.

[5+5]





8.a) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find this relationship.

b) Discuss the maxima and minima of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. [5+5]

OR

9 a) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$; $y = e^{2t} \cos 3t$; $z = e^{2t} \sin 3t$. Find $\frac{du}{dt}$ as a total derivative and verify the result by direct substitution.

b) Find Maxima and Minima of $x^3 y^2 (1-x-y)$. [5+5]

10a) Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$.

b) Evaluate $I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. [5+5]

OR

11 a) By changing the order of integration, evaluate $\int_0^1 \int_{x^2}^{2-x} xy dy dx$.

b) Using Triple Integral, Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$. [5+5]

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