R22

Code No: 181AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, January/February - 2025

MATRICES AND CALCULUS

(Common to EEE, CSE, IT, CSIT, CSE(CS), CSE(DS), CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 7\\ 3 & 6 & 10\end{bmatrix}$. [1]

b) Write the condition for the consistency of linear systems $AX=B$, consisting of m linear equations involving n unknowns. [1]

c) If $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$, then find the Eigen values of A^2. [1]

d) State the Cayley-Hamilton Theorem. [1]

e) Find c of Lagrange's mean value theorem for $f(x)=e^x$ in $[0,1]$. [1]

f) Prove that $\int_0^\infty e^{-x^2}\, x^{2n-1}\, dx=\frac{1}{2}\Gamma(n)$. [1]

g) Find the first order partial derivatives of $z=x^3+y^3-3axy$. [1]

h) If $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$. [1]

i) Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. [1]

j) Evaluate $\int_0^1\int_1^2\int_2^3 (xyz)\,dzdydx$. [1]

PART - B

(50 Marks)

2.a) Find the inverse of the matrix $A=\begin{bmatrix}1 & 3 & 3\\1 & 4 & 3\\1 & 3 & 4\end{bmatrix}$ by Gauss-Jordan Method.

b) Discuss the consistency of the following system of equations $2x+3y+4z=11$, $x+5y+7z=15$, $3x+11y+13z=25$. If found consistent, solve it.

[5+5]

OR

3 a) Determine the general solution for the following homogeneous systems: $2x + y + z = 0,\ 4x + 2y + z = 0,\ 6x + 3y + z = 0,\ 8x + 4y + z = 0.$

b) Solve the following system by Gauss-Seidel method: $83x+11y-4z=95$, $7x+52y+13z=104$, $3x+8y+29z=71$. [6+4]

4.a) Find the Eigen values and Eigen vectors of $A=\begin{bmatrix}3 & 1 & 4\\0 & 2 & 6\\0 & 0 & 5\end{bmatrix}$.

b) Using Cayley-Hamilton theorem, obtain the inverse of the matrix $\begin{bmatrix}-1 & -1 & -2\\8 & -11 & -8\\-10 & 11 & 7\end{bmatrix}$.

[5+5]

OR

5. Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ to canonical form by orthogonal transformation. Determine the index, signature and nature of the quadratic form. [10]

6 a) Verify Rolle's theorem for the function $f(x)=(x+2)^3(x-3)^4$ in $[-2,3]$.

b) Verify Cauchy's mean value theorem for the functions $f(x)=\log_e x$ and $g(x)=\frac{1}{x}$ in the interval $[1,e]$. [5+5]

OR

7 a) Using the Lagrange mean value theorem, show that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3}-\frac{1}{8}$.

b) Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^4}}=\frac{1}{4}\beta\left(\frac{1}{4},\frac{1}{2}\right)$. [5+5]

8.a) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find this relationship.

b) Discuss the maxima and minima of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. [5+5]

OR

9 a) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$; $y = e^{2t}\cos 3t$; $z = e^{2t}\sin 3t$. Find $\frac{du}{dt}$ as a total derivative and verify the result by direct substitution.

b) Find Maxima and Minima of $x^3 y^2 (1-x-y)$. [5+5]

10a) Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$.

b) Evaluate $I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$. [5+5]

OR

11 a) By changing the order of integration, evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$.

b) Using Triple Integral, Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$. [5+5]

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